GCE AS/A level

0985/01

MATHEMATICS S3
STATISTICS 3

P.M. FRIDAY, 22 June 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S12-0985-01)

1. Three numbers are chosen at random **without replacement** from the set {1, 2, 3, 4, 5}. Determine the sampling distribution of

(a) the mean of the three chosen numbers, [5]

(b) the median of the three chosen numbers. [2]

2. The manager of a factory that manufactures a certain type of string claims that its mean breaking strength is 100 Newtons. In order to test this claim, the breaking strengths, in Newtons, of a random sample of ten pieces of this string were determined with the following results.

97·1 100·3 98·6 97·7 101·2 97·6 98·9 101·1 98·5 99·3

You may assume that this is a random sample from a normal distribution with mean μ and variance σ^2.

(a) Calculate unbiased estimates of μ and σ^2. [4]

(b) (i) State suitable hypotheses for testing the manager's claim using a two-sided test.

(ii) Carry out an appropriate test at the 5% significance level. Giving a reason, state your conclusion in context. [7]

3. Alan plays a certain game on his computer and he wants to estimate the probability p of winning. He plays the game 120 times and wins 54 of these games. It may be assumed that successive games are independent.

(a) (i) Calculate an unbiased estimate of p.

(ii) Determine an approximate 95% confidence interval for p. [6]

(b) Brenda also plays this game and she decides to determine a 90% confidence interval for the probability of her winning a game. She therefore plays the game n times and wins x of these games. She correctly calculates an approximate 90% confidence interval to be

[0·455, 0·581]

where the confidence limits are given correct to three decimal places.
Determine

(i) an unbiased estimate of the probability that Brenda wins a game,

(ii) the value of n,

(iii) the value of x. [7]

4. A motoring organisation wishes to compare the fuel consumption of two car models A and B. It therefore sets up a test in which 50 cars of each model are each supplied with 5 litres of fuel and are driven at a predetermined speed along a track until the fuel is used up. Let x denote the distance (in miles) travelled by each car of model A before stopping and let y denote the distance (in miles) travelled by each car of model B before stopping. The results are summarised below.

$$\sum x = 2565, \quad \sum x^2 = 131659, \quad \sum y = 2590, \quad \sum y^2 = 134232$$

(a) State suitable hypotheses to investigate whether or not there is a difference between the mean distances travelled by model A and model B cars when given 5 litres of fuel. [1]

(b) Calculate the p-value of the above results and interpret your value in context. [10]

5. The temperature y°C in an oven x minutes after switching on the oven can be assumed to satisfy the equation $y = \alpha + \beta x$. In order to estimate α and β, the following measurements were made.

x	0	1	2	3	4	5
y	20·0	34·4	49·3	65·6	79·7	96·5

(a) Calculate least squares estimates for α and β. [8]

(b) The values of x are exact but the values of y are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·75. Determine a 99% confidence interval for β. [5]

TURN OVER

6. The probability density function of the continuous random variable X is given by

$$f(x) = \frac{2x}{a^2} \qquad \text{for } 0 \leqslant x \leqslant a,$$
$$f(x) = 0 \qquad \text{otherwise,}$$

where a is an unknown positive constant.

(a) Obtain an expression for $E(X)$ and show that

$$\text{Var}(X) = \frac{a^2}{18}.$$ [7]

(b) In order to estimate a, a random sample of n observations of X is taken.

(i) The mean of the observations in the sample is denoted by $\overline{X}$. Find the value of the constant c such that

$$U = c\overline{X}$$

is an unbiased estimator for a and obtain an expression for the variance of U.

(ii) Let Y denote the largest observation in the sample. You are given that

$$E(Y) = \frac{2na}{2n+1} \quad \text{and} \quad \text{Var}(Y) = \frac{na^2}{(n+1)(2n+1)^2}.$$

Find the value of the constant d such that

$$V = dY$$

is an unbiased estimator for a and obtain an expression for the variance of V.

(iii) Show that

$$\frac{\text{Var}(U)}{\text{Var}(V)} = \frac{n+1}{2}.$$

State, with a reason, which is the better estimator, U or V. [13]